Seminar on Case Studies in Operations Research (Mat-2.4177)

Dependent Evidence in Probabilistic Reasoning - Applying Copulas in Combining Non-Destructive Tests of Final Disposal Canisters

VTT, Posiva

Backlund, Ville-Pekka
Piironen, Juho
Tuovila, Henri (Project manager)
Wolf, Sebastian

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Aalto University
School of Science

Contents

1 Introduction

This project work is performed on the course “Seminar on Case Studies in Operations Research” in spring 2012. The actual research problem was set by VTT, and carried out in cooperation with VTT and Posiva.

This report discusses subjects that relate to ensuring the safety of final disposal of nuclear waste in Finland. In Finland, one of the responsible companies for final disposal of nuclear waste is Posiva, and it has worked together with VTT with subjects that relate to the safety assessment and reliability of the final disposal canisters. In this project new approaches have been developed to cope with the dependence of various tests that are carried out in order to assess the safety of these final disposal canisters.

One of the major concerns for the licensee responsible of disposing nuclear waste is that what is the probability for a defective canister to end up burying to the bedrock, despite of all the tests and studies for the canisters. The evidence to assess the safety of the canister is gathered by carrying out different tests, for both the interior of the weld in between canister and the lid, and to the surface of the lid. Posiva has defined the critical levels of intact copper in the weld, and all the canisters need to fulfill these minimum requirements [1]. One of the general problems in this project has been the difficulty to combine evidence mathematically from different tests in order to define the total probability of not detecting a defective canister.

One assumption would be to assume all the tests to be independent of one another. This might, however, lead to too optimistic probabilities of not detecting a defect. For dependent tests, some part of the information they produce is already contained in the information some other test produces, and thus does not improve the knowledge about the underlying matter as much as independent tests would. The aim of this report is to study a method for taking into account the different possible dependence structures of the testing methods.

2 Background

2.1 Final disposal of nuclear waste in Finland

In Finland nuclear power companies are responsible for nuclear waste management [2]. Therefore, in 1995, the two operating nuclear power plant companies Teollisuuden Voima Oyj and Fortum Power and Heat Oy established Posiva Oy as an expert organization responsible for their spent nuclear fuel

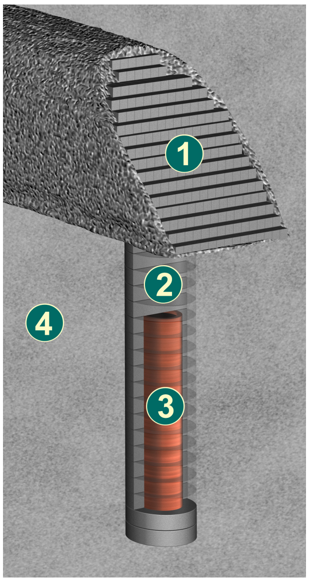

Figure 1: A figure of the final disposal canister after placing it into the repository. The numbers in the figure describe: 1. tunnel backfill, 2. bentonite, 3. final disposal canister, 4. bedrock.

[2] generated in the four reactors in Eurajoki and Loviisa and also for three more reactors which have been planned to be built in the future.

The aim of Posiva is to find and to implement a way to encapsulate and store the nuclear fuel which is not harmful for any organic nature and to ensure that it will remain intact for very long time due to the low decay rate of certain nuclear waste.

In order to realize this it is planned to store the spent nuclear fuel in the bedrock of Olkiluoto in 400m depth, packed in copper canisters [2]. The first canisters shall be stored in 2020 and the whole installation will have a capacity of 4500 canisters. The Olkiluoto installation shall take the fuel of the seven nuclear reactors mentioned above for the following 100 years. Then the tunnels will be sealed mainly by bentonite and bedrock. The final disposal canister in the final repository is seen in figure 1.

One of the safety issues in this context is the quality of the copper canisters. After a canister is filled with nuclear waste its lid has to be welded onto the body. This process is fully remote controlled due to the nuclear radiation. The weld might be a weak point since different defects can occur in it. By Posiva's definition of a safe canister at least 35 mm of intact copper must protect the nuclear fuel. To ensure that the weld fulfills this condition, it is investigated by four different testing methods [3], where two methods enable one to investigate the interior of the specimen while the other two search for defects on and close to the surface. If one of these four testing methods detects a defect, further investigations and measurements will be done in order to decide whether the defect is acceptable or the canister has to be rejected. The latter case should be avoided since the canisters are very expensive in relation to the application of the testing methods. The tests are non-destructive which means that a specimen does not have to be destroyed in order to test it. Non-destructive testing (NDT) methods ensure that the measurements can be repeated if necessary.

2.2 Non-destructive testing

The four testing methods used in NDT are [1]:

Visual testing (VT) The first method applied to the weld of a copper canister. Due to the nuclear radiation the testing is remote controlled. The camera is installed above the canister and it searches the surface for notches and cracks.

Eddy current (EC) EC testing uses electromagnetic induction to detect defects in conductive materials. It is possible to detect holes and also changes of material properties on the copper surface and up to 10 mm below it. The purpose of this method is to quantitatively determine the location and shape of any defect or internal structure within the copper.

Radiography testing (RT) In RT method x-rays are sent to penetrate solid matter. Using the fact that the x-rays are differently absorbed by different material, information about the density and thickness of the material can be derived. This is useful for finding holes in the interior of the copper and also to determine their volumes.

Ultrasonic testing (UT) UT uses high frequency sound energy to produce sound waves and to make measurements. Ultrasonic inspection can be used for defect detection, evaluation, dimensional measurements and material characterization.

Thus VT and EC are the surface testing methods and RT and UT can be used to study the interior of the weld.

The data obtained from NDT is used to assess whether each canister is acceptable. Figure defining the acceptance and rejection process is seen in figure 2. Complete explanation of the process is out of the scope of this report, but one critical point is already seen from the figure: if a canister

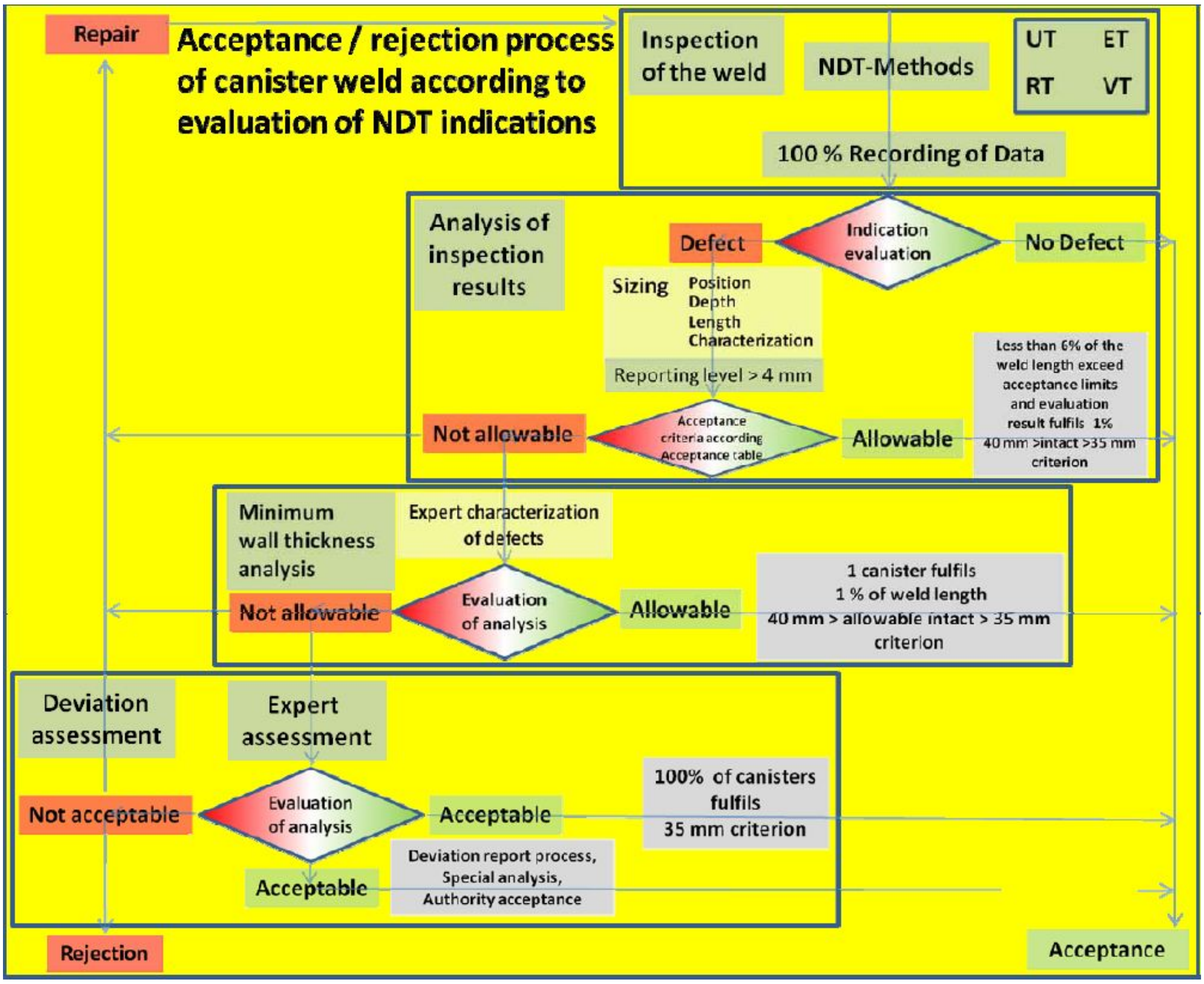

Figure 2: Acceptance and rejection process of canister weld according to evaluation of NDT indications [1]

passes through the *Indication evaluation* diamond in the upper right corner, it is accepted and placed to the final repository. Canister is accepted if none of the NDT methods indicate a possible defect in it. Therefore, studying the probability of all the methods simultaneously failing to detect a defect is important.

2.3 Probability of Detection curves

The data gathered from NDT-tests is processed and built up to so called Probability of Detection (POD) curves. These curves present the probability of detecting a defect of a certain size. They also provide the confidence inter-

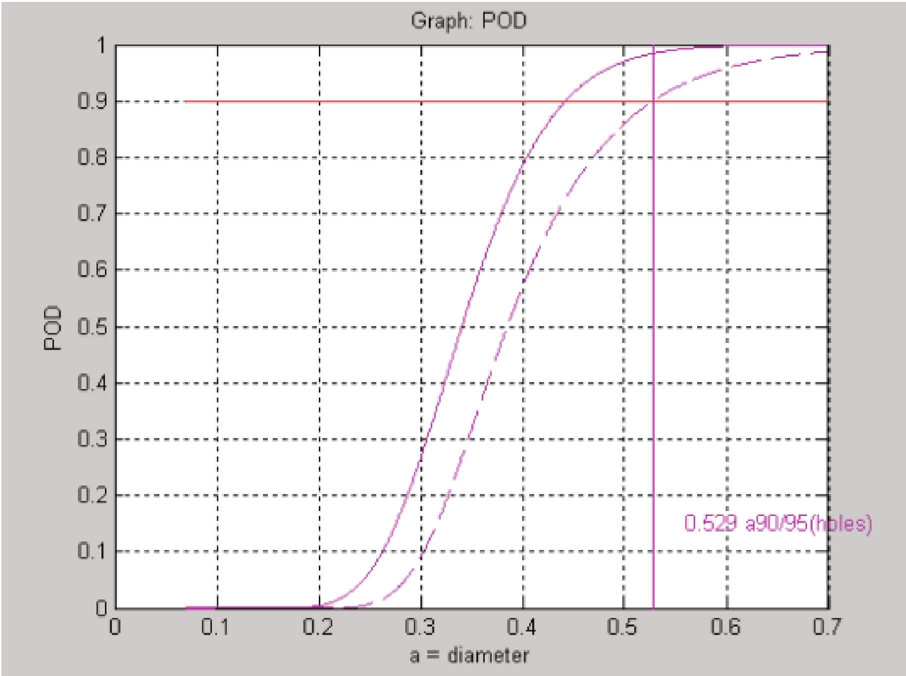

Figure 3: An example of a POD curve. The figure presents the POD for high frequency eddy current testing for holes. The probability of detection and its confidence interval is on the y-axis, whereas the size of the hole on the y-axis.

vals for the detection probabilities. POD curves are widely used in industry as a tool for reliability assessment [4].

The POD curves are constructed from the NDT data by either *â vs a* or *hit/miss* methods. In the first, the physical response signal of a test method (â) is measured for known defect sizes (a). In the latter, the data available is just discretely valued to *hit* (defect is detected) or *miss* (no detection). The precise explanation of the construction of the POD curves is outside the scope of this report, but for a comprehensive review see [4].

One POD curve is just for one specific NDT method. If a NDT method is used with different settings, for example eddy current with high or low frequency, a POD for each of setting is produced. The PODs are also dependent on the type of the defect, so every defect type has its own POD. In the case of eddy current testing for the final disposal canisters a total of four POD curves are constructed using two different frequencies and two defect types (notch and hole) [3]. Some preliminary POD curves for NDT methods from artificial defects are already constructed by Posiva, but from now on our reasoning assumes that all the POD curves utilized would be available.

3 Coping with the dependencies

One general way to assess the dependence between random variables is so called correlation coefficient, which is widely used in time series analysis. Other possibilities include assuming certain multidimensional distributions, but eventually that approach would as well lead to assuming a certain level of correlation.

One easy assumption that could be made here is the assumption of either independence, or complete dependence. In the case of our study, the assumption of independence would mean that the tests detect defects independently according to their own uncertainties of detecting a defect. The calculation of expected joint probability of two independent random variables is presented in equation 1.

$$E[X \cdot Y] = E[X] \cdot E[Y] \tag{1}$$

On the other hand, the assumption of total dependence would essentially mean that the other test is useless, as the other one already gives same results as the other would give. In this sense, performing multiple tests that are completely dependent would be waste of time and other resources. The best case would be negative correlation between the test results, so that the tests would be complementary. This case could indeed be true if for example other test is better in detecting defects of certain geometric shape, when the other constantly misses these geometrical shapes.

In equation 2 is shown the calculation of joint expectation of dependent random variables in general case, which can be derived starting from the definition of covariance:

$$E[X \cdot Y] = E[X] \cdot E[Y] + Cov[X, Y]. \tag{2}$$

Basically, if the correlation between random variables is negative, the covariance will also be negative. For positively correlated random variables then positive covariance indicates in that sense higher expected value for the joint value 2 than in the independent case 1. By this reasoning, dependent NDTs would imply higher probabilities for not detecting the defect, than in the case of independence.

Next, the concept of copula functions is introduced. Copulas are nowadays a popular approach when dealing with dependent random variables, especially in the field of finance. [5]

3.1 Short introduction to copulas

The generic way to combine dependent probability distributions into a joint distribution is to use so called copula functions. Copula functions have been around for many years already (since 1959, first introduced by Sklar [6]). In this report the possibility of using copula functions to describe the dependence structure between different NDT methods is studied.

The basic idea of copula functions is simple. They are used to describe the dependence structure of different random variables. When dealing with copulas in the context of NDT-methods, the marginal distribution for each NDT in the each point estimate of the POD curve are supposed to be known. These marginal distributions can then be combined using copula functions in order to estimate an expected value. The multidimensional distribution contains the information about both: the marginal distributions and the dependence structure. In order to study only the dependence structure, one solution is to separate it from the multivariate distribution and then get the actual copula.

The basic idea of copulas [7] starts with a random vector $(X_1, X_2, .., X_d)$, where X_is are random variables, and the system is d-dimensional. The cumulative distribution function for the distribution of each random vector can be written $F_i(x) = P(X_i \leq x)$. Now by performing the transformation for the random vector by utilizing the individual marginal distribution functions, following uniformly distributed random vectors are in hand:

$$(U_1, U_2, ..., U_d) = (F_1(X_1), F_2(X_2), ..., F_d(X_d)) \tag{3}$$

Now the copula C is defined by the joint distribution of uniformly distributed random vector $\mathbf{U}$:

$$C(u_1, u_2, ..., u_d) = P(U_1 \leq u_1, U_2 \leq u_2, ..., U_d \leq u_d). \tag{4}$$

Copula essentially contains the same information as the joint probability distributions, except the information of marginal distributions. If the marginal distributions are known, as well as the copula function, the same can be done inversely, namely:

$$F(x_1, x_2, ..., x_d) = C(F_1(x_1), F_2(x_2), ..., F_d(x_d)). \tag{5}$$

The equation 5 is also known as *Sklar's theorem* [6], and it can be used to create multi dimensional joint distribution for separate random variables, assuming that their dependence structure (copula) and individual marginal distributions can be assessed some other way.

Copulas are usually defined as cumulative distribution functions of the uniform margins, but in practice they are commonly differentiated into density form, which is more convenient and can express graphically more easily the dependence structure. If the copula is sufficiently well-behaving, and differentiable, the copula density function can be expressed as follows:

$$c(u_1, u_2, ..., u_d) = \frac{\partial_1 \partial_2 \ldots \partial_d}{\partial u_1 \partial u_2 \ldots \partial u_d} C(u_1, u_2, ..., u_d). \tag{6}$$

As the empirical data is usually presented graphically as scatter plots, this formulation of copula density function enables us to compare theoretical graphical presentations to empirical (or simulated) scatter plots.

Some most commonly used copula functions are presented next.

3.1.1 Independence copula

Independence copula is probably the simplest copula one could imagine. It is defined by the following equation:

$$\Pi(u_1, u_2, ..., u_d) = u_1 \times u_2 \times \cdots \times u_d. \tag{7}$$

Utilizing the independence copula formulated in 7 is essentially the same as multiplying probabilities of certain random variables together. Independence copula is a trivial case, and applied also in this report.

3.1.2 Comonotonicity copula

Another example of a very simple copula is so called comonotonicity copula, which indicates perfect positive dependence between random variables. It is defined as follows:

$$M(u_1, u_2, ..., u_d) = \min(u_1, u_2, ..., u_d). \tag{8}$$

3.1.3 Countermonotonicity copula

The counterpart for comonotonicity copula is naturally the countermonotonicity copula, which indicates the other extreme case: perfect negative dependence between random variables. In two-dimensional case it is defined:

$$W(u_1, u_2) = \max(u_1 + u_2 - 1, 0) \tag{9}$$

3.1.4 Gaussian copula

Some copula functions can be derived directly from joint distributions. One of these is so called Gaussian copula, which comes from the multinormal distribution. Gaussian copula for two random variables is defined:

$$C_\rho^{Ga}(u_1, u_2) = \Phi_\Sigma(\Phi^{-1}(u_1), \Phi^{-1}(u_2)). \tag{10}$$

In equation 10 Φ_Σ is the cumulative density function of bivariate normal distribution with zero mean, and covariance matrix Σ, and Φ^{-1} is the inverse cumulative distribution function of normal distribution, in other words it is the quantile function of normal distribution.

Some other examples of copulas, which are derived from multi-dimensional distributions, exist. For example so called t-copula, which comes from multi-dimensional student t-distribution. These are however not covered in this report, as they are not used in later parts.

3.1.5 Archimedean copulas

The copulas that fall into family of Archimedean copulas are defined generally:

$$C(u_1, u_2, ..., u_d) = \psi(\psi^{-1}(u_1), \psi^{-1}(u_2), ..., \psi^{-1}(u_d)). \tag{11}$$

In the equation 11 above, the function ψ is called a generator function, and for most Archimedean copulas the generator function also needs one or more parameters as an input, which then define the copula more specifically. Archimedean copulas are not derived from any distributions, but they are defined explicitly and they are easy to express mathematically.

Generator functions for most interesting copulas are for Gumbel copula

$$\psi_G(t) = e^{-t/\theta} \tag{12}$$

for Clayton copula

$$\psi_C(t) = (1+t)^{-1/\theta} \tag{13}$$

and for Frank copula

$$\psi_F(t) = -\frac{\ln(1-(1-e^{-\theta})e^{-t})}{\theta}. \tag{14}$$

The parameter for each copula function is θ, which takes values in the sequence $(0, \infty)$ for Clayton and Frank copulas, and $[1, \infty)$ for Gumbel copula. Their properties are highly dependant on the value of the parameter, but some general properties can be assessed here: Frank copula indicates symmetric tail dependence, and the values in between are less dependent. Gumbel copula indicates greater dependence to the upper tail of the distributions, when the Clayton copula to the lower tail.

3.2 Defining the approach

Next, an approach for utilizing the copulas in studying the possible dependencies in the evidence of a defected final disposal canister is presented.

A defective canister is accepted if no indication of a defect is obtained, that is, all the n testing methods fail. Thus, our main interest is the joint probability $q(s)$ of not detecting a defect of size s in any of the NDT tests. The probability of a single method failing can be derived straight from the POD curve

$$q_i(s) = 1 - \mathrm{POD}_i(s), \quad i \in \{1, \cdots, n\}, \tag{15}$$

where i describes the index of NDT method and $\mathrm{POD}_i(s)$ the probability of detecting a defect of size s, taken as a point from a corresponding POD curve.

The uncertainty of the point estimate in the POD curve must be taken into account. This is considered with the help of Beta distributions. Betas are continuous probability distributions defined in the interval (0,1), thus they are suitable for presenting the probability distribution of the detection probability. A Beta(α, β) is fitted to the point estimate taken from the POD curve so that the mean of the beta corresponds to the point estimate of the probability of detecting a defect of certain size and the $\mathrm{p}_{0.05}$ point of the beta corresponds to the 95% confidence interval the point estimate.

The cumulative distribution functions of the beta distributions represent now the marginal distributions in equation (5). After **assuming** a copula C to describe the dependence structure of the tests for a certain defect size and indicating the cumulative beta function for a probability of detection of an NDT method i with B_i, the joint cumulative probability of detecting a defect of size s can be calculated as

$$P(x_1, \cdots, x_n|s) = C(B_1(x_1), \cdots, B_n(x_n)). \tag{16}$$

P is conditional to the size of the defect s, because the beta functions represent the probability of detection and its uncertainty for a particular s.

Then, to get the joint probability density $p(x_1, \cdots, x_n|s)$, the joint cumulative is differentiated according to the equation (6). This probability density is then used to calculate the ultimate probability of all the methods failing at the same for the same canister:

$$q(s) = \int_0^1 \cdots \int_0^1 (1 - x_1) \cdots (1 - x_n) p(x_1, \cdots, x_n|s) \mathrm{d}x_1 \cdots \mathrm{d}x_n \tag{17}$$

Visual examination helps to understand the idea of the approach. Consider two beta distributions, Beta(5,25) and Beta(7,20), whose probability density

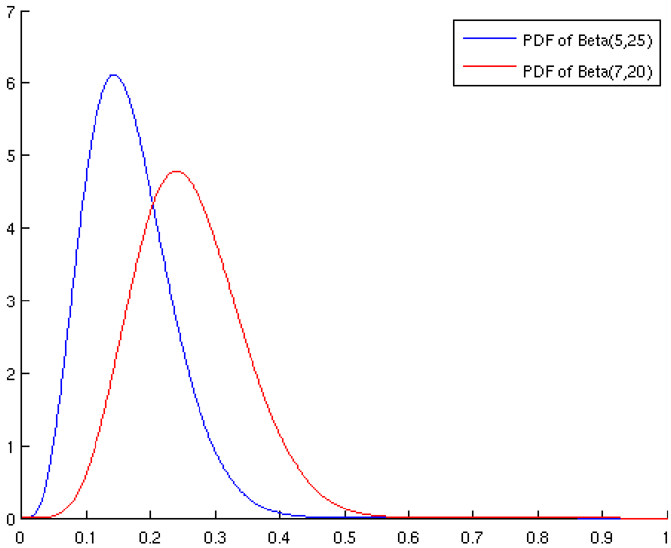

Figure 4: Probability density functions of two example Betas.

functions can be seen in figure 4. Then choose a gaussian copula with correlation parameter $\rho = 0.0$, which defines the variables to be independent. The cumulative and probability density functions of the copula can be seen in the upper row of the figure 5. The probability density is flat because the variables are independent. This copula does not change the joint distribution of the betas seen in the lower row of figure 5.

By choosing another gaussian copula with $\rho = -0.8$, the copula produces a complex dependence structure between the variables, seen in the upper right corner of figure 6. Use of this copula alters also the joint probability density of the betas to follow the shape of the copula, as can be seen in the lower right corner of the figure 6

3.3 Choosing the copula

The comonotonicity and countermonotonicity copulas define the upper and lower bounds within every other copula is [7] so they, along with the independence copula are the natural choices for copulas to study.

As the most trivial copulas represent only the extreme cases of dependence, the approach is expanded to cover also the Gaussian copula. By exploiting the Gaussian copula the intensity of dependence can be varied by modifying the correlation coefficient parameter of the copula.

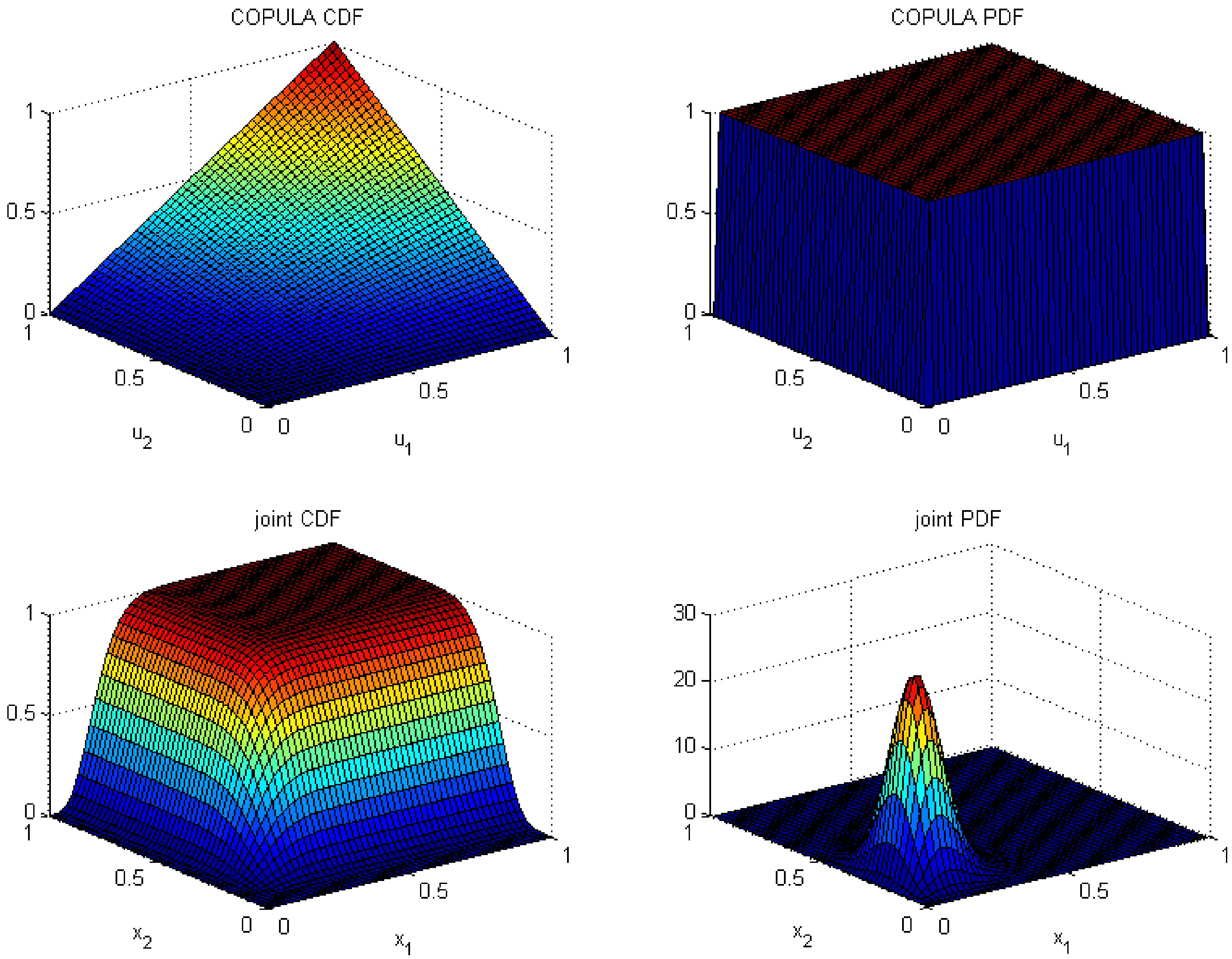

Figure 5: Clockwise from upper left corner: CDF of gaussian copula with $\rho = 0.0$, PDF of gaussian copula with $\rho = 0.0$, CDF of joint beta distribution, PDF of joint beta distribution.

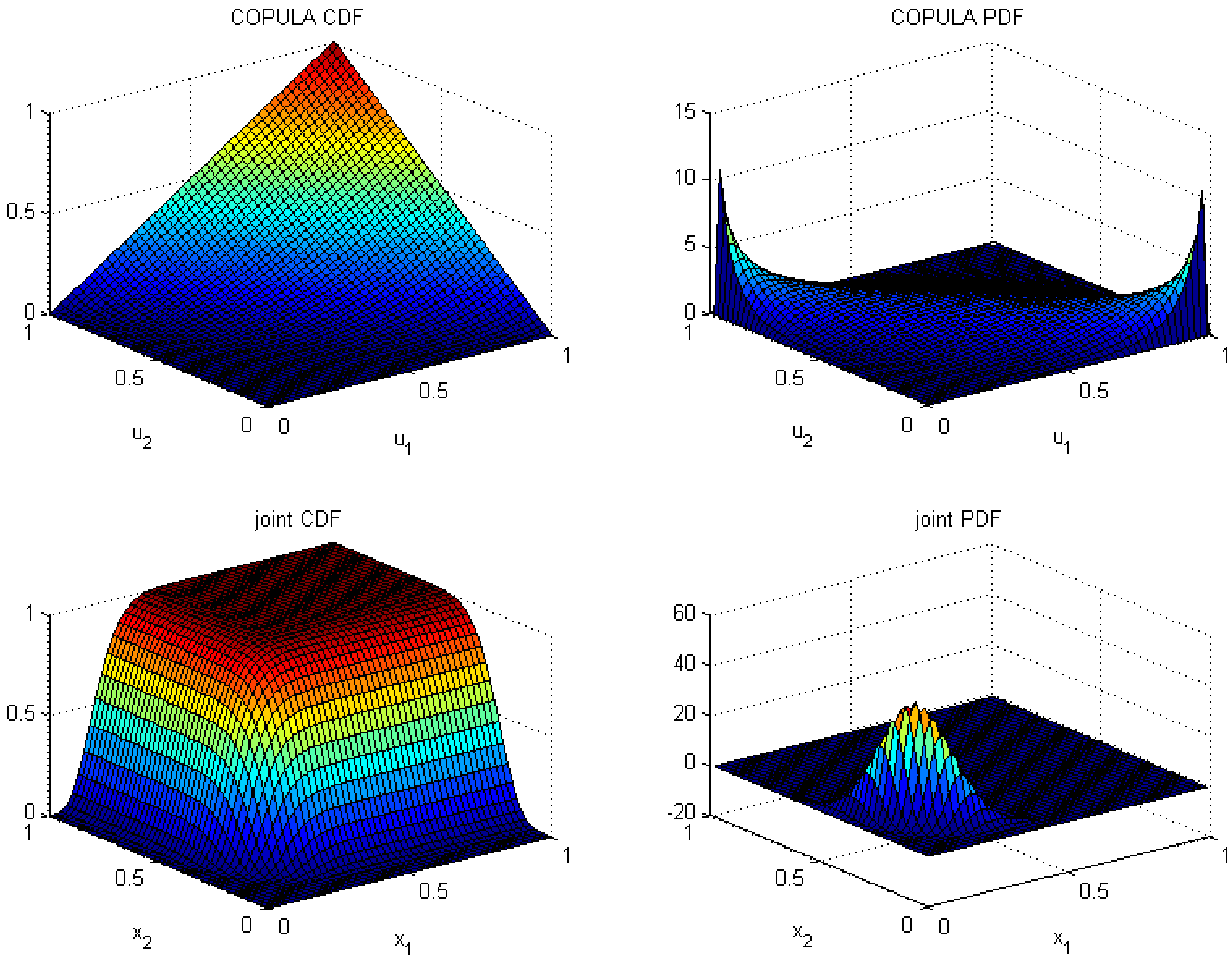

Figure 6: Clockwise from upper left corner: CDF of gaussian copula with $\rho = -0.8$, PDF of gaussian copula with $\rho = -0.8$, CDF of joint beta distribution, PDF of joint beta distribution.

Table 1: Parameters for the beta distributions utilized in this study.

Method	Parameter α	Parameter β	$E[x]$
Visual testing	10.5	0.5	0.9545
Eddy current testing	12.5	0.5	0.9615

3.4 A case study of the approach

Next a case study of the approach is presented. It is reasonable to assume that the surface methods (VT,ET) are quite independent of the volumetric methods (RT,UT), thus only the probability of not detecting a defect with just the surface methods is considered.

Above defined approach considers the defect with just its size. In reality, the geometry of a defect varies greatly and is measured in multiple metric dimensions. Also, the position of defect in the weld has an effect on the severity of it. Currently, there is not much information on the distribution of sizes and positions of real defects. Thus, the case is limited to study only one critical defect given by the experts of Posiva [3]:

a) **through hole of diameter 0.5 mm** If a lid has a through hole of sufficiently large diameter all inspection methods (VT, ET, UT and RT) should give an indication of it. However, none of them can detect the complete information about the through hole and thus results of different inspection methods need to be combined in the inference process. The diameter of the reference hole is close to the resolution limit of UT and RT, and thus an indication made by VT and ET can be important, too.

A defect of this type forms a severe risk if positioned in the final disposal cave.

The estimates for Beta distributions for a defect type a are assumed to be the same as in [3] where they are obtained from POD curves or reasoned otherwise. This gives us the parameters seen in table 1. The mean value of the distribution corresponds to the point estimate of the POD.

The distribution functions generated by parameters in table 1 are presented in figure 7. From this figure it can be seen that the probability mass is highly concentrated on the latter end of the sequence $(0, 1)$ and that the beta-functions are almost the same for ET and VT. This means that the probability of detection is relatively high, since the expected value of the betas correspond to the expected value of the probability of detection in this specific point estimate.

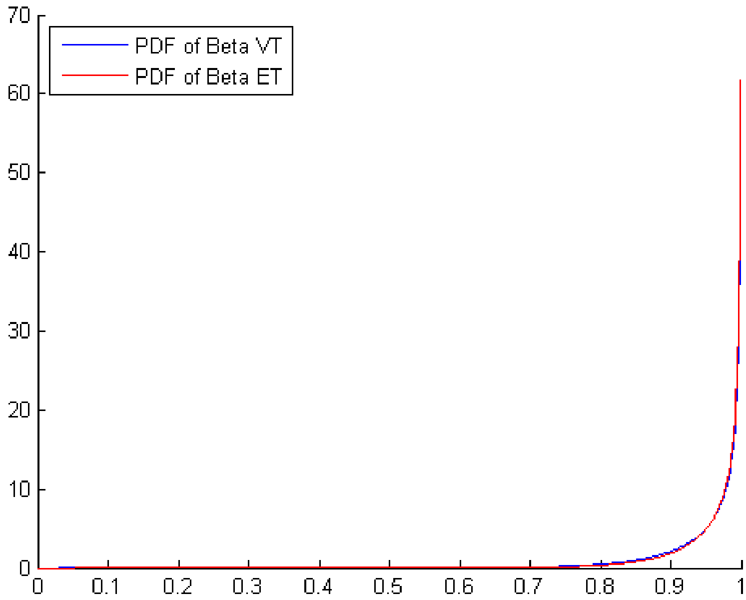

Figure 7: The beta-functions utilized in this project.

Table 2: Estimates for q for different simple copulas.

Copula	q
countermonotonicity	0.000528
independence	0.001803
comonotonicity	0.004939

Since the equation (17) cannot be solved analytically, Monte Carlo integration [8] is used. The integral of equation (17) for two tests can be calculated numerically in the form

$$q(s) = \frac{1}{N}\sum_{i=1}^{N}(1-x_1)(1-x_2)p(x_1,x_2|s), \tag{18}$$

where N is the number of sample rounds used for MC and both x_1 and x_2 are individual samples from uniform distribution on the interval $(0,1)$. In this case N = 100000.

Following the above defined procedure and choosing first comonotonicity, countermonotonicity, independence copula and gaussian copula with correlation parameter $\rho \in (-1,1)$, the estimates for q seen in figure 8 are obtained. The values for the simple copulas are seen also in the table 2.

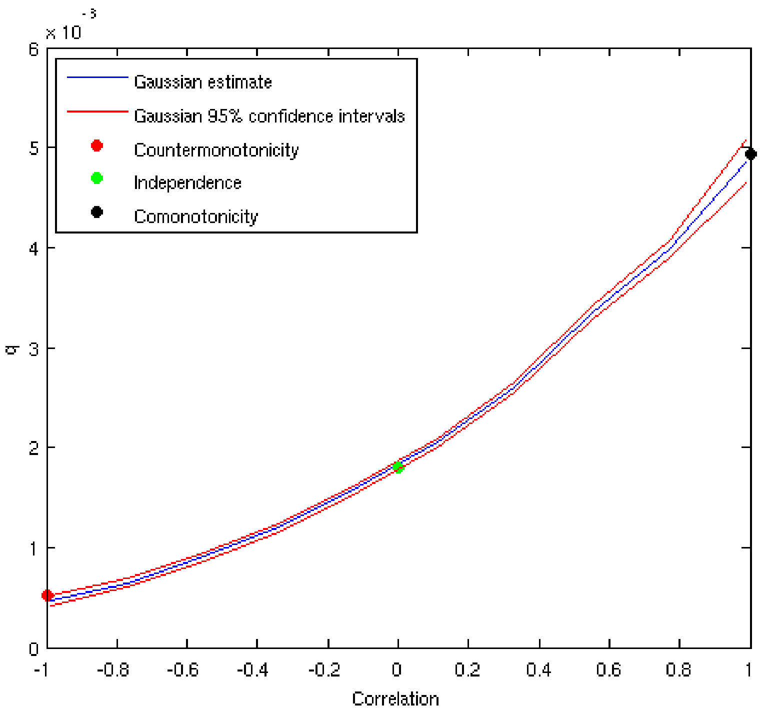

Figure 8: Estimates for q as the copula is varied. The line shows the gaussian copula as a function of correlation ρ with the 95% confidence interval of the mean value based on Monte Carlo integration. The dots show the countermonotonicity, independence and comonotonicity copula. Their position is respective to the type of correlation they represent.

The first remark from the figure is that the simple copulas match with the values of Gaussian copulas, when placed according to the correlation values they present. This was an expected outcome. Another remark is that the probability of failed detection increases monotonically with the correlation, which is also intuitively clear behaviour especially when $\rho > 0$. For the negative correlations the interpretation of the result is that the testing methods become complementary, i.e. if the first method fails to detect the defect then the probability of detecting it with the other method increases. This is possible for defects of certain geometry which are likely to be missed with one testing method but likely to be spotted with another one.

It is also of interest to do some quantitative analysis between the different dependencies. The probability of missing with both methods is about $5/2$ larger in case of complete dependence compared to the case of total independence. This coefficient tells the magnitude of the error made when falsely assuming independence between the testing methods. With assumed number of 2800 final disposal canisters [2], this means $2800(q_{\text{dependence}} - q_{\text{independence}}) \approx 8.8$ defective and not detected canisters more in the final repository.

3.5 Validating the copula model

So far the study has concentrated in comparing different dependence structures, their effect on the joint probability distributions and their expected values. The real world problem would then be: how to validate the use of certain copula? Empirically copula models are easy to exploit for example in the field of finance, where data is not scarce. In safety assessment, however, the data available is not yet sufficient to determine the dependence structure credibly. On the other hand, also creating some new data for validation purposes would require much effort and might be costly.

If the question of effort and cost is neglected, we might consider making artificial defects, which would then be studied by different NDT methods in order to estimate the dependence. Artificial defects are used already when deriving the POD curves, so this should be possible in practice. Let say, we want to estimate a reasonable copula for two uncertainty distributions, namely beta distributions, which are the uncertainties of detection probabilities for the specific defect size. One way to do it would be to make a large number of artificial defects of certain size, which would all then be inspected by both methods. On the other hand, having huge amount of hit-or-miss points would not give much information about the dependence.

The data used here should perhaps be something more sophisticated than just binary valued pairs. In real life the data would be, instead of hit-and-miss, something more complex, for example amplitudes of electric current from the measurement. This kind of non-binary data could then be interpreted in order to make the judgement to accept or reject the canister. Here, instead of interpreting the data into hit-or-miss, we could perhaps be able to construct a set of paired points indicating the intensity of evidence. The interpretation of this intensity could then be probabilistic, and after a certain intensity threshold, the existence of a defect in a sample would be evident.

4 Conclusions

Studies carried out in this project work consider the possibility for utilizing copula functions to take into account the dependence structure of different non-destructive testing methods in the safety assessment of nuclear waste final disposal canisters. Earlier studies have been done by assuming the testing methods to be either independent or coupled [3], but in this study the perspective is extended into several different dependence structures.

One of the main findings is the effect of dependence on the probability estimate of not detecting a defect in a final disposal canister when combining evidence from multiple testing methods. It is shown that the level of dependence clearly affects the results. According to this study, incorrect assumptions for the dependence structure of the tests might lead to erroneous decisions and too optimistic estimates. If the dependence between tests increases from zero correlation into correlation of $\rho = 0.5$, the probability of not detecting a defect increases by more than 50 %.

The study also shows that making the different methods comprehensive (negatively correlated) would improve the results. For example in NDT sense this would mean that testing the weld with one method from multiple angles is reasonable, if the detection depends on the geometry of the defect.

It is worth noticing that the case studied in this report was just for demonstrating the approach of utilizing copulas to model the dependence structure of the evidence. The case covered only two of the four tests, and only one critical defect defined by the experts of Posiva. Therefore the numerical values obtained are only for comparing various dependencies and not to be used as such.

Drawbacks for the approach utilizing copulas is that the dependence structure between different random variables is not easily validated. So far there has been no data that could have been used to validate the dependence structure, so the studies in this paper remain in the level of speculating the effects of different dependencies on the end result: the probability of not detecting a critical defect.

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Table 3: The risks of the project

Issue	Status
Ambiguous starting point	Realized
Personal workload	Not realized
BBN is not applicable	Not realized (not covered)
Team member quits	Not realized
Difficulties in schedule	Realized
Suggested methods are not usable	Not known yet

A Appendix 1: Comments on the course

The ambiguous starting point caused much difficulties for the group, as the final goals were clarified not even before the mid-term report. The final report ended up being relatively theoretical, and not so much about quantitative issues, which was caused by the lack of data. First half of the project was more about studying the subject, and all promising topics related to it. Most of the topics studied were out of the scope of the final report, so they did not end up being utilized.

All in all, our group worked well together and major difficulties were avoided, although the group faced a slight rush in the end. Everyone contributed well, and finding time for meetings was not as difficult as it seemed in the beginning. Any of the risks indicated in the project plan and mid-term report were not realized in their most severe form, but it is a matter of opinion whether this project provided VTT what they were expecting. Probably at least some new ideas for VTT were introduced, but it is of course possible that none of these is usable in practice. The risks of the project are presented In table 3. As it can be seen in table 3, the BBN approach introduced in mid-term report and project plan did not end up being covered in the final report, as the group thought it would be better to cover certain areas more deeply rather than listing topics and covering them superficially.

As the project was in the end more about theory and less practical, dividing tasks for group members was difficult. Planning the schedule in advance was also difficult or even impossible, because it was not possible to divide tasks into smaller entireties. Dividing workload for group members was more allocating reading for group members and finding topics that one or two members could familiarize themselves deeper. Working in group was not really efficient, as the group held many meetings in which everyone participated. Working together was still essential, and was rewarded by small advances one at a time. In group meetings the group went through the subjects that had recently been read, and then negotiate which of these seems most promising.

In terms of the original assignment, we clearly took into deeper study the question of handling dependencies between dependent evidence. Other subjects which were indicated in the original assignment were not under deep study in the final report. Perhaps collecting the work of the first half of the project could have been presented in a form of a literary review, which was however not expected from VTT. Moreover, it would have been much less work for the group, if for example the assignment would have had a clear literary list to start with. As no one of the group members was an expert on issues related to probabilistic reasoning, acquiring a basic knowledge of the issue was essential, but also laborious.

The project in general offered a unique viewpoint to see applying mathematical methods in practice to real life problems. Practical issues turned out to be more complex than expected, and many simplifications to the practical problem were to be made in order to address it mathematically. It also turned out that real life problems tend to have no simple correct solution, rather solutions that are better than others from certain aspect but worse from other aspects.